

For Reference

NOT TO BE TAKEN FROM THIS ROOM

THESIS
1955(F)
#15

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

CAT. No. 1935

LOWE-MARTIN CO. LIMITED

EX LIBRIS UNIVERSITATIS ALBERTAENSIS





Digitized by the Internet Archive
in 2018 with funding from
University of Alberta Libraries

<https://archive.org/details/park1955>

THE UNIVERSITY OF ALBERTA

- I. A QUANTITATIVE STUDY OF THE ENERGIES OF SINGLE
ALPHA PARTICLES FROM THE PHOTONUCLEAR REACTIONS
OF SILVER AND BROMINE NUCLEI

AND

- II. A QUALITATIVE STUDY OF MULTIPLE ALPHA EVENTS
INDUCED BY PHOTONUCLEAR REACTIONS OF CARBON,
NITROGEN AND OXYGEN NUCLEI

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

FACULTY OF ARTS AND SCIENCE

DEPARTMENT OF PHYSICS

by

YOON SOO PARK, B.Sc.,

EDMONTON, ALBERTA,

SEPTEMBER, 1955.

Acknowledgement

I wish to express my gratitude to Dr. D. B. Scott, for his constant assistance and guidance during the progress of this work, and also to Mr. L. H. Greenberg, Department of Physics, Regina College, University of Saskatchewan, for making possible and suggesting this research.

I further wish to thank the other members of the Department of Physics for their help and encouragement.

I am grateful to the University of Alberta, whose financial assistance has made this research possible.

TABLE OF CONTENTS

	Page
ABSTRACT.....	1
INTRODUCTION.....	2
1. Single Track Events	
2. Two Track Events	
3. Three Track Events	
4. Four Track Events.	
THEORY.....	6
1. Parallel and Cascade Photo-alpha Reactions.	
2. Threshold-favored Alpha Particle Emission.	
EXPERIMENTAL METHOD.....	9
1. Plate Irradiation	
2. Plate Development	
3. Plate Survey.	
EXPERIMENTAL RESULTS.....	12
DISCUSSION.....	34
BIBLIOGRAPHY.....	36
APPENDIX I Calculation of Range and Energy.	37
APPENDIX II Calculation of Escape Correction Factor.....	40
APPENDIX III Identification of The Momentum Balance Among The Three Prongs of A Carbon Star.....	41

ABSTRACT

An Ilford E1 nuclear research emulsion was exposed to 600 roentgens of bremsstrahlung from the University of Saskatchewan betatron at maximum betatron energy 24 Mev. The plate was developed by a grain-gradation process in order to suppress proton tracks and background fog. The double-peaked single alpha particle energy spectrum resulting from the photo-disintegration of silver and bromine nuclei was studied. Interpretations of the energy spectrum of these events were deduced. Multiple-alpha events induced by photonuclear reactions of carbon, nitrogen and oxygen nuclei were also studied.

INTRODUCTION

Photonuclear reactions which involve the emission of particles from the nuclei provide a method for studying nuclear structure. Nuclear research emulsions exposed in the beam of the gamma-rays have shown tracks of alpha particles resulting from the photo-disintegration of nuclei present in emulsions. The nuclear events observed in an emulsion can be grouped according to the number of tracks which the emitted alpha particles have left in the emulsion.

1. Single Track Events.

Most of these events are attributed to alpha particles resulting from the photo-disintegration of silver and bromine in nuclear emulsions. Nabholz, Stoll and Waeffler at Zurich (1), working with 17.6 Mev. Lithium gamma-rays, have reported work on these events.

The energy spectrum of single alpha particles exhibits a characteristic double peak. A typical spectrum (Fig. 1a) shows two energy peaks. Haslam and co-workers at the University of Saskatchewan (2), and Millar and Cameron (3), interpreted the energy distribution in terms of the statistical theory of photonuclear reactions and attributed the higher energy peak to parallel and cascade photo-alpha reactions of the types (γ, α) and $(\gamma, \alpha n)$ in silver and bromine, and the lower energy peak to threshold^{old}-favored alpha particle emissions in reactions of the types $(\gamma, \pi \alpha)$, $(\gamma, p \alpha)$ and $(\gamma, \gamma' \alpha)$.

Greenberg (5), while working at the University of Saskatchewan, obtained a slightly different pattern of the energy spectrum of single alpha particles, which showed statistically small but definite maxima in the region above 6 Mev, between the two characteristic peaks. One would not expect such

maxima from alpha particles originating in silver and bromine. However, if these maxima do exist, they could be due to alpha particles arising in the nuclei of light elements such as carbon, nitrogen and oxygen. Moreover, in this case, it is probable that at least a small fraction of the high-energy peak is due to alpha particles originating in a light nucleus. Also most of the low-energy peak may perhaps be attributed to a (γ, α) reaction in a light nucleus.

The main purpose of this project has been to resolve these secondary maxima as reported by Greenberg. For this purpose 2693 single alpha particle tracks have been measured. An attempt was made to interpret the energy spectrum in terms of the statistical theory of nuclear reactions.

2. Two-Track Events.

These are events occurring in light nuclei. These events have been reported and interpreted as the reactions $N^{14}(\gamma, \alpha)B^{10}$, $O^{16}(\gamma, \alpha)C^{12}$ and $C^{13}(\gamma, \alpha)Be^9$ by Millar and Cameron (6). An alpha particle ejected from a light nucleus gives a considerable amount of energy to the recoiling nucleus. In many cases the track of the recoil nucleus is visible, as a short, densely-ionized stub at the beginning of the alpha particle track.

In the case of a carbon nucleus emitting an alpha particle another possible reaction is $C^{12}(\gamma, \alpha)Be^8$. However, this interpretation of the observed events is considered unfavorable in view of the lifetime of Be^8 (3).

3. Three-Track Events.

These events occur mainly in C^{12} . The most probable reactions by which the C^{12} nucleus might break into three-pronged alpha particle stars are

- (a) $C^{12}(\gamma, 3\alpha)$
- (b) $C^{12}(\gamma, \alpha) Be^8; Be^8 \rightarrow 2 He^4$
- (c) $C^{12}(\gamma, \alpha) Be^{8*}; Be^{8*} \rightarrow 2 He^4$

Since Haenni et al. (7) originally interpreted these events as proceeding through the excited state of Be^8 , i.e., the reaction (c), the interpretation of these events has been confirmed and identified by many workers, such as Teledgi and Eder (8), Goward and Wilkins (7), and Miller and Cameron (3). If the events show a momentum balance, the reaction is of the type (a). The events due to the reactions (b) and (c) have a long single alpha particle track with a narrow V formed by the Be^8 or Be^{8*} nucleus break-up.

Greenberg (5) located a fairly large number of the events due to the reaction type (b) and is presently examining the angular distribution of the first-emitted alpha particle tracks to determine the spin of Be^8 .

There are the events which do not show a momentum balance among three α prongs. These events have been identified as due to the reaction $N^{14}(\gamma, Li^6)2He^4$ by Wilkins and Goward (10).

Besides the above events, the reported three track events are $C^{13}(\gamma, 2\alpha)He^5$ (5),

$Ag(\gamma, 2\alpha)Ru$ (11) (identity uncertain), $(Ru \xrightarrow{\beta} Tc)$
 $Ag(\gamma, 2\alpha)Tc$ (3) (identity uncertain).

4. Four-Track Events.

These events occur in oxygen. Goward and Wilkins (12) attributed the majority of the cases to the reactions $O^{16}(\gamma, 4\alpha)$ and $O^{16}(\gamma, 2He^4)Be^8$;

$\text{Be}^8 \rightarrow 2 \text{He}^4$. Other possible processes in oxygen are

(a) $\text{O}^{16}(\gamma, 2\text{Be}^8)$; $2 \text{Be}^8 \rightarrow 4\text{He}^4$: Goward & Wilkins (13)

(b) $\text{O}^{16}(\gamma, 2\text{Be}^{8*})$; $2 \text{Be}^{8*} \rightarrow 4\text{He}^4$

(c) $\text{O}^{16}(\gamma, 2 \text{He}^4)\text{Be}^{8*}$; $\text{Be}^{8*} \rightarrow 2 \text{He}^4$

(d) $\text{O}^{16}(\gamma, \text{He}^4)\text{C}^{12*}$; $\text{C}^{12*} \rightarrow \text{He}^4 + \text{Be}^{8*}$ $\text{Be}^{8*} \rightarrow 2 \text{He}^4$

The problem remains to identify the exact mechanism of these reactions.

Goward (14) and Millar and Cameron (3) have reported events which are probably due to the reaction in nitrogen, $\text{N}^{14}(\gamma, \text{D})3\text{He}^4$

THEORY

Photonuclear excitation of medium-heavy and heavy nuclei, including silver and bromine takes place usually by means of an electric dipole interaction between photons and nuclei (15). Such interaction may result principally in the direct excitation of a nuclear proton (16). If the proton fails to escape before the excitation energy is spread among the nucleons an excited compound nucleus is thereby formed.

1. Parallel and Cascade Photo-alpha Reactions.

When an excited compound nucleus decays initially by emission of a photon or one of several different particles, it is said that the different modes of decay are in parallel competition with one another. When the residual nucleus is left in an excited state further decay may occur by one of several modes. Then it is said that there is cascade competition between the modes of the first decay and the modes of the second decay.

The emission of alpha particles from the compound nucleus in parallel competition in a single decay is known to be greatly favored. According to the statistical theory of photonuclear reactions (4, 17), the energy distribution of alpha particles from a compound nucleus is

$$I(E_\alpha)dE_\alpha = \text{const. } E_\alpha \sigma(E_\alpha) \omega_R(\epsilon) dE_\alpha,$$

where E_α is the alpha particle energy, $\sigma(E_\alpha)$ is the cross section of the residual nucleus for capture of an alpha particle with energy E_α , and $\omega_R(\epsilon)$ is the level density of the residual nucleus at an energy ϵ . If the compound nucleus was excited by a photon of energy $h\nu$, then

$$\epsilon = h\nu - E_\alpha - Q_\alpha,$$

where Q_α is the binding energy of the alpha particle in the original nucleus. Millar and Cameron (3) attributed the higher energy alpha

particle peak to such (γ, α) reactions in silver and bromine.

The residual nucleus will occasionally be left sufficiently excited to emit further particles after the emission of an alpha particle from the highly excited compound nucleus. Hence initial alpha particle emission may be followed by particle emission in cascade reactions of the types ($\gamma, \alpha n$), ($\gamma, \alpha \alpha$) and ($\gamma, \alpha p$). In the neighborhood of silver the binding energies of alpha particles, protons and neutrons are respectively 3, 7 and 9 Mev. When the excitation energy of the residual nucleus is less than 9 Mev it is not possible for a neutron to be emitted. In this case charged-particle emission may occur. Such events are said to be threshold-favored. They are discussed in the next section. The emission of neutrons will be more probable, since the secondary emission of a charged particle is impeded by the presence of the Coulomb barrier. Therefore, the reaction ($\gamma, \alpha n$) may also provide a main source of high energy alpha particles.

The emission of an alpha particle may occur in cascade reactions of the types ($\gamma, \gamma' \alpha$) and ($\gamma, \alpha \gamma'$). In such reactions initial photon absorption is followed by the electric dipole radiation of a low energy gamma-ray (18). Hence, these reactions will also provide a source of high energy alpha particles.

2. Threshold-favored Alpha Particle Emission.

In the special case where the residual excited nucleus emits a charged particle, the emission of such particles is termed threshold-favored. Haltsam et al, (2) have explained the lower energy alpha particle peak in terms of threshold-favored alpha particle emission in silver and bromine. Cross sections for the nuclear absorption of photons of energy less than 10 Mev

are very small, so that reactions of any sort induced by photons of energy less than 10 Mev are not expected to be important. The probability of cascade events increases with increasing photon energy for energy above 10 Mev. The excited compound nucleus which has absorbed higher energy photons may decay initially by emission of a particle or a photon and leave the residual nucleus with an excitation in excess of the binding energy of an alpha particle but not of a proton or a neutron. Hence, in this case, the emission of an alpha particle is the only mode of de-excitation by which the residual nucleus may lose its energy of excitation. Low energy alpha particles may, therefore, be expected to be emitted in threshold-favored cascade reactions of the types $(\gamma, \gamma' \alpha)$, $(\gamma, n \alpha)$, $(\gamma, p \alpha)$, and $(\gamma, \alpha \alpha)$.

At low excitation energies only alpha particles and gamma-rays will be emitted from an intermediate residual nucleus. The emission of alpha particles will become more probable as the excitation energy is increased. But at excitation energies above 7 Mev protons can also be emitted and the alpha particle yield will be relatively reduced. At still higher energies, electric dipole radiation may lower the intermediate nuclear energy to the threshold-favored region. This may produce tertiary reactions of such types as $(\gamma, n \gamma' \alpha)$, $(\gamma, p \gamma' \alpha)$. It may be expected that threshold-favored alpha particle emission will be most probable when a residual nucleus is not quite sufficiently excited to emit a proton.

EXPERIMENTAL METHOD.

(The work of this project has been concerned with the examination of a plate already exposed and developed at Saskatoon. Comments on the exposure and development are included here for the sake of completeness).

1. Plate Irradiation.

Ilford type E1 nuclear emulsions, 1" x 3" were exposed to an irradiation corresponding to 600 roentgens of bremsstrahlung from the University of Saskatchewan betatron operating at maximum energy 24 Mev. To get significant data on the energy distribution of single alpha particle tracks as well as other events it is necessary to locate a very large number of events. Searching for the events is very time-consuming. An increase in the dose given to the plates in this experiment compared with other workers (2, 3) resulted in an increase in the density of events. After development an emulsion showed very distinct alpha particle tracks in a light background fog. The background fog is due to the secondary electrons from emulsion, film covering, air, donut wall, shield, etc. More radiation ^{could} be put into the plates if the sources of the secondary electrons could be removed. Plates were exposed so that the direction of the beam was along the surface of the emulsion.

2. Plate Development.

Ilford type E1 nuclear emulsions are comparatively less sensitive to lightly-ionized particles such as protons or electrons and very sensitive for recording alpha particles of fairly low energy. The plates were developed in a modified Van der Grinten's grain gradation developer (26). This process suppressed secondary electron fog as well as proton tracks.

3. Plate Survey.

Among developed plates, a plate E1(205) was selected and studied under a Bausch and Lomb binocular microscope with a nominal magnification of 1000 diameters. The plate showed alpha tracks very distinctly in the presence of the slight background fog. Faint lightly-ionized proton tracks were occasionally visible, but were easily distinguishable from alpha particles.

Total area of 1.65 cm.^2 on the plate was searched. All the events observed were carefully located and recorded and the energies of all single alpha particle tracks were measured. To obtain the energies of the particles it is necessary to measure track lengths (Appendix I) and have knowledge of a range - energy relationship in the emulsion. Cameron, while working at Chalk River, calculated a range-energy relation for Ilford type E1 emulsions. (Table IV). As a result of work by numerous observers, for example (24), it is believed that this relation is valid within 2%.

During observation it was noticed that there were tracks which escaped from the emulsion and some tracks which ended within 2 microns of either surface of the emulsion and could not be distinguished from those which escaped. Therefore it was necessary to take into account an escape correction for the observed tracks. (Appendix II and Table V).

It also was discovered that some tracks were shorter and denser than common alpha particle tracks. These were regarded as tracks of particles with charge greater than two. Energy and momentum considerations make it possible to determine whether the particle was of charge two or greater. However, all doubtful tracks were assumed to be due to alpha particles. This assumption seems not unreasonable, considering the fact that the emission of alpha particles is much more probable than the emission of heavier particles, because of their lower threshold energies and Coulomb barrier heights.

1944-1945

The first part of the report deals with the general situation in the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The second part of the report deals with the economic situation in the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The third part of the report deals with the social situation in the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The fourth part of the report deals with the political situation in the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The fifth part of the report deals with the cultural situation in the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The sixth part of the report deals with the military situation in the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The seventh part of the report deals with the foreign relations of the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The eighth part of the report deals with the internal affairs of the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The ninth part of the report deals with the future of the country during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country. The tenth part of the report deals with the conclusion of the report during the year 1944-1945. It is a very interesting and detailed account of the events of that year, and it is a very valuable contribution to the history of the country.

In order ^{to} interpret the experimental observations and to assign reasonable reaction mechanisms, it was necessary to know the atomic composition of the emulsion. This, for type E1 emulsion as supplied by Ilford Limited, is shown in Table I.

The results for the single alpha-particle events are shown in Figure 1a in which the number of events in each 0.5 Mev interval is plotted as a function of energy. It is worth noting that the limit of resolution in this work is set, not ~~so much~~ ^{only} by the measurement of track length and particle energy, ^{but} ~~as~~ by the number of tracks measured.

EXPERIMENTAL RESULTS

Numbers of events found in 1.65 cm^2 in the plate are shown in Table II.

1. Single Track Events

Figure II shows a picture of a single alpha particle track observed in the plate. This was the most common event observed. These events are due to the ejection of alpha particles from silver and bromine nuclei which are the principal heavy nuclei present in the emulsion. Tracks of the recoiling residual nuclei were not observed in the emulsion, while they were observable in the case of the light elements emitting alpha particles.

The energy spectrum of alpha particles from silver and bromine nuclei is shown in Table III and Figure Ia. A total of 2693 alpha particle tracks is plotted as a function of energy in each 0.5 Mev interval, and the spectrum has also been corrected for the escape of alpha particles from the emulsion.

The energy spectrum shows two peaks, one at about 4.75 Mev and one at about 9.25 Mev.

Millar (19) calculated the alpha particle spectrum by the nuclear evaporation theory which is applicable to middle-weight nuclei excited by

γ -rays. The theoretical energy spectrum for the plate exposed at maximum energy 24 Mev is shown in Figure Ib for the purpose of comparison with the experimental results. The theoretical curve is based on the assumption that simple (γ, α) reactions account for the majority of the alpha particle tracks observed. It will be noted that there is rough agreement between the observed and calculated higher energy peak as to peak position, and limit of particle energy. However, on the other hand, the theoretical curve shows few alpha particles with energy less than 6 Mev and therefore offers no explanation for the lower energy peak observed in the region 4 Mev to 5 Mev.

According to statistical theory, the higher energy peak will be attributed to alpha particles emitted in parallel and cascade reactions of the types (γ, α) , $(\gamma, \alpha n)$ and $(\gamma, \gamma' \alpha)$, and the lower energy peak to threshold-favored cascade reactions of the types $(\gamma, \gamma' \alpha)$, $(\gamma, n \alpha)$ and $(\gamma, p \alpha)$. The reactions $(\gamma, \alpha \alpha)$ were observable as two-pronged alpha particle stars in this experiment, and therefore the events were classified as two track events. The energy spectrum does not show the secondary maxima between the two energy peaks as observed by Greenberg.

2. Two Track Events.

There were a considerable number of two track events which could not be regarded as due to the ejection of alpha particles from oxygen, nitrogen and carbon 13, and recoil of the residual nuclei. Both tracks showed two easily-identifiable alpha-particle tracks. These are probably due to the ejection of two alpha particles from silver and bromine.

Though the track of the recoiling nucleus was visible as a short stub at the origin of the alpha particle tracks, the number of these events observed would be dependent mainly on the person searching since the range of the recoiling nucleus is less than 2 microns. Also it was often difficult to distinguish between a bent single alpha particle track and an event which involved two alpha particles.

3. Three Track Events.

The disintegration of carbon into three alpha particles accounted for most of the three particle stars. When a carbon atom is excited by absorption of a photon it may be stabilized by emission of any of several particles. If an alpha particle is emitted the residual nucleus will be

Be^8 , which may be in either the ground state or in an excited state. Be^8 is unstable and disintegrates with a very short lifetime into two alpha particles. On the other hand, a carbon nucleus may disintegrate into three alpha particles without passing through the intermediate Be^8 nucleus.

Of 736 observed stars 514 were tentatively identified as the reaction $\text{C}^{12}(\gamma, 3\alpha)$ and 124 proceeded through an excited or ground state of Be^8 . Figure III shows a picture of a typical carbon star which is probably due to the reaction as proceeding through the intermediate Be^8 nucleus. The longest alpha particle track proceeding outward from the centre is due to the first-emitted alpha particle and the narrow V-configuration is regarded as due to Be^8 break-up. If there was no V-configuration among the prongs, the star was attributed to the reaction which proceeds without passing through either an excited or ground state of Be^8 .

For a number of stars the lengths and angles of the three prongs were measured. When the momenta of the prongs were added, the total momentum balance was shown to be zero with a very small correction for the photon momentum within the limit of error. The energy of the photon involved in the reaction was obtained by adding the kinetic energies of the three alpha particles and the binding energy of 7.15 Mev. (Appendix III),

There were a number of stars which did not show a momentum balance among the three prongs. This is probably the single stage three-body breakup of nitrogen described by the equation $\text{N}^{14}(\gamma, \text{Li}^6) 2 \text{He}^4$. Figure IV shows a picture of such a nitrogen star, a Li^6 nucleus leaving a short stub in the emulsion.

1. The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation $f(x) = \int_0^x f(t) dt$. It is shown that $f(x)$ is a constant function, and its value is determined by the initial condition $f(0)$.

2. In the second part, we consider the problem of finding the maximum value of the function $f(x)$ on the interval $[0, 1]$. It is shown that the maximum value is attained at $x = 0$, and its value is $f(0)$.

3. The third part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation $f(x) = \int_0^x f(t) dt$. It is shown that $f(x)$ is a constant function, and its value is determined by the initial condition $f(0)$.

4. In the fourth part, we consider the problem of finding the maximum value of the function $f(x)$ on the interval $[0, 1]$. It is shown that the maximum value is attained at $x = 0$, and its value is $f(0)$.

4. Four Track Events.

There were 60 four pronged alpha particle stars, of which 37 were attributable to the reaction $O^{16}(\gamma, 4\alpha)$ and 23 to the reaction $O^{16}(\gamma, 2\alpha)Be^8$; $Be^8 \rightarrow 2 He^4$. Figure V shows a picture of four-pronged oxygen stars due to the former reaction which exhibits no other obvious relation among the four prongs except conservation of linear momentum. However Figure VI shows the narrow V-configuration between two of four prongs which may be interpreted as due to the Be^8 breakup into two alpha particles.

The nitrogen four-pronged stars due to the reaction $N^{14}(\gamma, D)^3He^4$ reported by Goward, and Millar and Cameron were not observed.

TABLE I

ATOMIC COMPOSITION OF ILFORD NUCLEAR RESEARCH EMULSTIO E1

Element	Ag	Br	I	C	H	O	N	S
Gm/cm. ³	1.85	1.34	0.052	0.272	0.056	0.266	0.067	0.010
Gm-atom/cc	0.0171	0.0167	0.00041	0.0226	0.056	0.0167	0.0048	0.00031

TABLE II

Numbers of Events found in 1.65 cm.² of The Plate E1(205)
Exposed at Maximum Betatron Energy 24 Mev.

Events	Numbers of Events/1.65 cm. ²
Single Track Events.....	2693
Ag(γ , α)Rh	
Br(γ , α)As	
Two Track Events.....	58
Three Track Events.....	736
C ¹² (γ , 3 α).....	514
C ¹² (γ , α)Be ⁸ \rightarrow 2 α)	
C ¹² (γ , α)Be ^{8*} \rightarrow 2 α).....	124
N ¹⁴ (γ , Li ⁸)2He ⁴	102
Four Track Events.....	60
O ¹⁶ (γ , 4 α).....	37
O ¹⁶ (γ , 2He ⁴)Be ⁸ \rightarrow 2 α).....	23

TABLE III

Energy spectrum of single alpha particles found in 1.65 cm.² of the plate E1 (205) exposed at maximum betatron energy 24 Mev. The spectrum has been corrected for the escape of alpha particles from the emulsion.

Energy Interval (Mev)	Intensity	Ce	Corrected Intensity
1.00 - 1.50	3		3.1
1.40	2	1.021	3.1
1.50 - 2.00	8		8.2
1.50	1	1.022	1.0
.70	3	1.027	3.1
.90	4	1.031	4.1
2.00 - 2.50	24		24.9
2.00	4	1.033	4.1
.20	4	1.038	4.2
.30	6	1.040	6.2
.40	10	1.041	10.4
2.50 - 3.00	62		65.2
2.50	4	1.044	4.2
.60	4	1.046	4.2
.70	6	1.049	6.3
.80	37	1.052	38.9
.90	11	1.054	11.6
3.00 - 3.50	123		130.5
3.00	11	1.056	11.6
.10	16	1.059	16.9
.20	52	1.061	55.2
.30	22	1.064	23.4
.40	22	1.066	23.4
3.50 - 4.00	182		195.7
3.50	48	1.069	51.2
.60	20	1.072	21.4
.70	28	1.075	30.2
.80	25	1.078	27.0
.90	61	1.080	65.9
4.00 - 4.50	261		285.0
4.00	39	1.082	42.2
.10	39	1.086	42.4
.20	76	1.089	83.0
.30	61	1.093	66.9
.40	46	1.097	50.5
4.50 - 5.00	358		393.5
4.50	103	1.100	113.0
.60	67	1.102	74.0
.70	49	1.106	54.3
.80	102	1.109	111.0
.90	37	1.113	41.2

TABLE

Summary of the results of the investigation of the effect of the concentration of the solution on the rate of the reaction between the solution and the solid substance.

Concentration of the solution, g/l.				Rate of the reaction, g/h.	
0.1				0.1	0.2
0.1				0.1	0.2
0.2				0.2	0.4
0.3				0.3	0.6
0.4				0.4	0.8
0.5				0.5	1.0
0.6				0.6	1.2
0.7				0.7	1.4
0.8				0.8	1.6
0.9				0.9	1.8
1.0				1.0	2.0

TABLE III - CONTINUED

Energy Interval (Mev)	Intensity	Ce	Corrected Intensity
5.00 - 5.50	186		210.7
5.00	39	1.116	45.4
.10	56	1.120	62.7
.20	30	1.123	33.8
.30	38	1.124	42.8
.40	23	1.131	26.0
5.50 - 6.00	149		170.6
5.50	32	1.136	36.4
.60	26	1.140	29.6
.70	30	1.144	34.4
.80	28	1.148	32.2
.90	33	1.152	38.0
6.00 - 6.50	127		148.0
6.00	21	1.156	24.2
.10	27	1.160	31.4
.20	26	1.164	30.4
.30	33	1.170	38.6
.40	20	1.175	23.4
6.50 - 7.00	115		137.2
6.50	25	1.180	29.5
.60	24	1.184	28.4
.70	16	1.190	19.0
.80	28	1.195	33.5
.90	22	1.214	26.8
7.00 - 7.50	113		136.1
7.00	30	1.204	36.2
.10	14	1.210	16.9
.20	29	1.215	35.2
.30	21	1.221	25.6
.40	19	1.227	22.2
7.50 - 8.00	112		139.5
7.50	29	1.234	35.8
.60	18	1.240	22.4
.70	31	1.246	38.6
.80	13	1.252	16.3
.90	21	1.259	26.4
8.00 - 8.50	96		122.5
8.00	25	1.265	31.6
.10	21	1.271	26.6
.20	12	1.277	15.3
.30	21	1.284	27.0
.40	17	1.290	22.0

TABLE - 191

1940		1941		1942		1943		1944		1945		1946		1947		1948		1949		1950		1951		1952		1953		1954		1955		1956		1957		1958		1959		1960		1961		1962		1963		1964		1965		1966		1967		1968		1969		1970		1971		1972		1973		1974		1975		1976		1977		1978		1979		1980		1981		1982		1983		1984		1985		1986		1987		1988		1989		1990		1991		1992		1993		1994		1995		1996		1997		1998		1999		2000		2001		2002		2003		2004		2005		2006		2007		2008		2009		2010		2011		2012		2013		2014		2015		2016		2017		2018		2019		2020		2021		2022		2023		2024		2025		2026		2027		2028		2029		2030		2031		2032		2033		2034		2035		2036		2037		2038		2039		2040		2041		2042		2043		2044		2045		2046		2047		2048		2049		2050		2051		2052		2053		2054		2055		2056		2057		2058		2059		2060		2061		2062		2063		2064		2065		2066		2067		2068		2069		2070		2071		2072		2073		2074		2075		2076		2077		2078		2079		2080		2081		2082		2083		2084		2085		2086		2087		2088		2089		2090		2091		2092		2093		2094		2095		2096		2097		2098		2099		2100		2101		2102		2103		2104		2105		2106		2107		2108		2109		2110		2111		2112		2113		2114		2115		2116		2117		2118		2119		2120		2121		2122		2123		2124		2125		2126		2127		2128		2129		2130		2131		2132		2133		2134		2135		2136		2137		2138		2139		2140		2141		2142		2143		2144		2145		2146		2147		2148		2149		2150		2151		2152		2153		2154		2155		2156		2157		2158		2159		2160		2161		2162		2163		2164		2165		2166		2167		2168		2169		2170		2171		2172		2173		2174		2175		2176		2177		2178		2179		2180		2181		2182		2183		2184		2185		2186		2187		2188		2189		2190		2191		2192		2193		2194		2195		2196		2197		2198		2199		2200		2201		2202		2203		2204		2205		2206		2207		2208		2209		2210		2211		2212		2213		2214		2215		2216		2217		2218		2219		2220		2221		2222		2223		2224		2225		2226		2227		2228		2229		2230		2231		2232		2233		2234		2235		2236		2237		2238		2239		2240		2241		2242		2243		2244		2245		2246		2247		2248		2249		2250		2251		2252		2253		2254		2255		2256		2257		2258		2259		2260		2261		2262		2263		2264		2265		2266		2267		2268		2269		2270		2271		2272		2273		2274		2275		2276		2277		2278		2279		2280		2281		2282		2283		2284		2285		2286		2287		2288		2289		2290		2291		2292		2293		2294		2295		2296		2297		2298		2299		2300		2301		2302		2303		2304		2305		2306		2307		2308		2309		2310		2311		2312		2313		2314		2315		2316		2317		2318		2319		2320		2321		2322		2323		2324		2325		2326		2327		2328		2329		2330		2331		2332		2333		2334		2335		2336		2337		2338		2339		2340		2341		2342		2343		2344		2345		2346		2347		2348		2349		2350		2351		2352		2353		2354		2355		2356		2357		2358		2359		2360		2361		2362		2363		2364		2365		2366		2367		2368		2369		2370		2371		2372		2373		2374		2375		2376		2377		2378		2379		2380		2381		2382		2383		2384		2385		2386		2387		2388		2389		2390		2391		2392		2393		2394		2395		2396		2397		2398		2399		2400		2401		2402		2403		2404		2405		2406		2407		2408		2409		2410		2411		2412		2413		2414		2415		2416		2417		2418		2419		2420		2421		2422		2423		2424		2425		2426		2427		2428		2429		2430		2431		2432		2433		2434		2435		2436		2437		2438		2439		2440		2441		2442		2443		2444		2445		2446		2447		2448		2449		2450		2451		2452		2453		2454		2455		2456		2457		2458		2459		2460		2461		2462		2463		2464		2465		2466		2467		2468		2469		2470		2471		2472		2473		2474		2475		2476		2477		2478		2479		2480		2481		2482		2483		2484		2485		2486		2487		2488		2489		2490		2491		2492		2493		2494		2495		2496		2497		2498		2499		2500		2501		2502		2503		2504		2505		2506		2507		2508		2509		2510		2511		2512		2513		2514		2515		2516		2517		2518		2519		2520		2521		2522		2523		2524		2525		2526		2527		2528		2529		2530		2531		2532		2533		2534		2535		2536		2537		2538		2539		2540		2541		2542		2543		2544		2545		2546		2547		2548		2549		2550		2551		2552		2553		2554		2555		2556		2557		2558		2559		2560		2561		2562		2563		2564		2565		2566		2567		2568		2569		2570		2571		2572		2573		2574		2575		2576		2577		2578		2579		2580		2581		2582		2583		2584		2585		2586		2587		2588		2589		2590		2591		2592		2593		2594		2595		2596		2597		2598		2599		2600		2601		2602		2603		2604		2605		2606		2607		2608		2609		2610		2611		2612		2613		2614		2615		2616		2617		2618		2619		2620		2621		2622		2623		2624		2625		2626		2627		2628		2629		2630		2631		2632		2633		2634		2635		2636		2637		2638		2639		2640		2641		2642		2643		2644		2645		2646		2647		2648		2649		2650		2651		2652		2653		2654		2655		2656		2657		2658		2659		2660		2661		2662		2663		2664		2665		2666		2667		2668		2669		2670		2671		2672		2673		2674		2675		2676		2677		2678		2679		2680		2681		2682		2683		2684		2685		2686		2687		2688		2689		2690		2691		2692		2693		2694		2695		2696		2697		2698		2699		2700		2701		2702		2703		2704		2705		2706		2707		2708		2709		2710		2711		2712		2713		2714		2715		2716		2717		2718		2719		2720		2721		2722		2723		2724		2725		2726		2727		2728		2729		2730		2731		2732		2733		2734		2735		2736		2737		2738		2739		2740		2741		2742		2743		2744		2745		2746		2747		2748		2749		2750		2751		2752		2753		2754		2755		2756		2757		2758		2759		2760		2761		2762		2763		2764		2765		2766		2767		2768		2769		2770		2771		2772		2773		2774		2775		2776		2777		2778		2779		2780		2781		2782		2783		2784		2785		2786		2787		2788		2789		2790		2791		2792		2793		2794		2795		2796		2797		2798		2799		2800		2801		2802		2803		2804		2805		2806		2807		2808		2809		2810		2811		2812		2813		2814		2815		2816		2817		2818		2819		2820		2821		2822		2823		2824		2825		2826		2827		2828		2829		2830		2831		2832		2833		2834		2835		2836		2837		2838		2839		2840		2841		2842		2843		2844		2845		2846		2847		2848		2849		2850		2851		2852		2853		2854		2855		2856		2857		2858		2859		2860		2861		2862		2863		2864		2865		2866		2867		2868		2869		2870		2871		2872		2873		2874		2875		2876		2877		2878		2879		2880		2881		2882		2883		2884		2885		2886		2887		2888		2889		2890		2891		2892		2893		2894		2895		2896		2897		2898		2899		2900		2901		2902		2903		2904		2905		2906		2907		2908		2909		2910		2911		2912		2913		2914		2915		2916		2917		2918		2919		2920		2921		2922		2923		2924		2925		2926		2927		2928		2929		2930		2931		2932		2933		2934		2935		2936		2937		2938		2939		2940		2941		2942		2943		2944		2945		2946		2947		2948		2949		2950		2951		2952		2953		2954		2955		2956		2957		2958		2959		2960		2961		2962		2963		2964		2965		2966		2967		2968		2969		2970		297	
------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	------	--	-----	--

TABLE III - CONTINUED

Energy Interval (Mev)	Intensity	Ce	Corrected Intensity
8.50 - 9.00	108		139.9
8.50	18	1.296	22.4
.60	25	1.305	32.5
.70	18	1.312	22.6
.80	21	1.318	27.8
.90	26	1.326	34.6
9.00 - 9.50	112		144.1
9.00	27	1.333	36.0
.10	31	1.341	41.6
.20	24	1.348	32.4
.30	18	1.354	24.4
.40	12	1.360	16.3
9.50 - 10.0	89		123.5
9.50	21	1.370	28.8
.60	25	1.380	34.6
.70	20	1.388	27.8
.80	15	1.396	21.0
.90	8	1.406	11.3
10.0 - 10.5	97		139.9
10.0	19	1.415	27.8
.1	14	1.423	19.8
.2	18	1.431	25.8
.3	26	1.442	37.5
.4	20	1.452	29.0
10.5 - 11.0	70		103.6
10.5	15	1.462	21.9
.6	17	1.472	25.0
.7	18	1.484	26.8
.8	13	1.496	19.4
.9	7	1.508	10.5
11.0 - 11.5	54		83.2
11.0	15	1.520	22.8
.1	11	1.530	16.8
.3	16	1.554	24.8
.4	12	1.567	18.8
11.5 - 12.0	65		104.4
11.5	10	1.574	15.7
.6	16	1.591	25.4
.7	10	1.605	16.1
.8	17	1.619	27.6
.9	12	1.633	19.6

TABLE 1 - 214

Date		Time		Location		Remarks	
1941	10/10	0800	0900	1000	1100	1200	1300
1941	10/11	0800	0900	1000	1100	1200	1300
1941	10/12	0800	0900	1000	1100	1200	1300
1941	10/13	0800	0900	1000	1100	1200	1300
1941	10/14	0800	0900	1000	1100	1200	1300
1941	10/15	0800	0900	1000	1100	1200	1300
1941	10/16	0800	0900	1000	1100	1200	1300
1941	10/17	0800	0900	1000	1100	1200	1300
1941	10/18	0800	0900	1000	1100	1200	1300
1941	10/19	0800	0900	1000	1100	1200	1300
1941	10/20	0800	0900	1000	1100	1200	1300
1941	10/21	0800	0900	1000	1100	1200	1300
1941	10/22	0800	0900	1000	1100	1200	1300
1941	10/23	0800	0900	1000	1100	1200	1300
1941	10/24	0800	0900	1000	1100	1200	1300
1941	10/25	0800	0900	1000	1100	1200	1300
1941	10/26	0800	0900	1000	1100	1200	1300
1941	10/27	0800	0900	1000	1100	1200	1300
1941	10/28	0800	0900	1000	1100	1200	1300
1941	10/29	0800	0900	1000	1100	1200	1300
1941	10/30	0800	0900	1000	1100	1200	1300
1941	10/31	0800	0900	1000	1100	1200	1300

TABLE III - CONTINUED

Energy Interval (Mev)	Intensity	Ce	Corrected Intensity
12.0 - 12.5	65		108.6
12.0	25	1.647	41.2
.1	6	1.651	9.9
.2	10	1.675	16.8
.3	15	1.691	25.4
.4	9	1.710	15.3
12.5 - 13.0	38		67.1
12.5	8	1.720	13.8
.6	9	1.749	15.7
.7	7	1.769	12.4
.8	7	1.790	12.5
.9	7	1.811	12.7
13.0 - 13.5	24		45.1
13.0	5	1.831	9.2
.1	5	1.853	9.3
.2	1	1.875	1.9
.3	12	1.900	22.8
.4	1	1.925	1.9
13.5 - 14.0	26		51.8
13.5	5	1.950	9.8
.6	9	1.975	17.8
.7	7	2.000	14.0
.8	3	2.020	6.1
.9	2	2.055	4.1
14.0 - 14.5	10		21.0
14.0	3	2.085	6.3
.1	4	2.116	8.2
.2	2	2.147	4.3
.3	1	2.181	2.2
14.5 - 15.0	3		6.9
14.5	1	2.247	2.2
.6	1	2.287	2.3
.8	1	2.360	2.4
15.0 - 15.5	3		7.7
15.0	1	2.447	2.4
.1	1	2.504	2.5
.4	1	2.675	2.8
15.5 - 16.0	3		8.7
15.6	1	2.850	2.9
.7	1	2.885	2.9
.8	1	2.920	2.9
16.0 - 16.5	2		6.2

TABLE III - CONTINUED

Energy Interval (Mev)	Intensity	Ge	Corrected Intensity
16.1	2	3.104	6.2
16.5 - 17.0	3		10.6
16.6	3	3.550	
17.0 - 17.5	1		1
18.5 - 19.0	1		1
Total Intensity	2693		3345

TABLE IV

RANGE-ENERGY RELATIONS FOR ALPHA PARTICLES IN ILFORD TYPE E1 EMULSIONS

R = range in microns
E = energy in Mev.

R → ↓ E →	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.00	0.01	0.02	0.03	0.04	0.05	0.07	0.08	0.10	0.12
1	0.15	0.18	0.21	0.24	0.27	0.30	0.34	0.38	0.42	0.45
2	0.49	0.52	0.55	0.58	0.62	0.65	0.68	0.72	0.76	0.80
3	0.84	0.87	0.91	0.95	0.99	1.03	1.07	1.10	1.14	1.18
4	1.21	1.24	1.27	1.31	1.35	1.38	1.42	1.46	1.49	1.53
5	1.56	1.59	1.62	1.66	1.70	1.73	1.76	1.79	1.83	1.86
6	1.89	1.93	1.96	1.99	2.02	2.04	2.07	2.11	2.14	2.17
7	2.20	2.23	2.26	2.28	2.31	2.34	2.37	2.39	2.42	2.45
8	2.48	2.50	2.53	2.55	2.58	2.60	2.63	2.66	2.68	2.72
9	2.74	2.76	2.78	2.81	2.84	2.86	2.88	2.92	2.94	2.96
10	2.98	3.01	3.03	3.05	3.07	3.10	3.12	3.15	3.17	3.20
11	3.22	3.25	3.27	3.29	3.32	3.34	3.37	3.39	3.41	3.43
12	3.46	3.48	3.50	3.52	3.54	3.56	3.58	3.61	3.63	3.65
13	3.67	3.70	3.72	3.74	3.76	3.78	3.80	3.82	3.84	3.86
14	3.88	3.91	3.93	3.95	3.97	3.99	4.01	4.03	4.05	4.07
15	4.09	4.11	4.13	4.15	4.17	4.19	4.21	4.23	4.25	4.27
16	4.29	4.31	4.33	4.35	4.37	4.39	4.41	4.43	4.45	4.47
17	4.48	4.50	4.51	4.53	4.55	4.57	4.58	4.60	4.62	4.64
18	4.66	4.67	4.69	4.71	4.73	4.75	4.77	4.78	4.80	4.82
19	4.84	4.85	4.87	4.89	4.91	4.93	4.95	4.96	4.98	5.00
20	5.02	5.03	5.05	5.07	5.08	5.10	5.12	5.13	5.15	5.17

TABLE IV - CONTINUED

R → ↓	E ↓	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
21		5.18	5.20	5.22	5.23	5.25	5.27	5.28	5.30	5.32	5.33
22		5.35	5.37	5.38	5.40	5.42	5.43	5.45	5.47	5.48	5.50
23		5.52	5.53	5.55	5.57	5.58	5.60	5.62	5.63	5.65	5.67
24		5.68	5.70	5.72	5.73	5.75	5.77	5.78	5.80	5.82	5.83
25		5.85	5.87	5.88	5.90	5.92	5.93	5.95	5.97	5.98	6.00
26		6.02	6.03	6.05	6.07	6.08	6.10	6.11	6.13	6.14	6.16
27		6.17	6.19	6.20	6.22	6.23	6.25	6.27	6.28	6.30	6.31
28		6.33	6.34	6.36	6.37	6.39	6.40	6.42	6.43	6.45	6.47
29		6.48	6.50	6.51	6.53	6.54	6.56	6.57	6.59	6.60	6.61
30		6.63	6.64	6.66	6.67	6.69	6.70	6.71	6.73	6.74	6.76
31		6.77	6.69	6.80	6.81	6.83	6.84	6.86	6.87	6.89	6.90
32		6.91	6.93	6.94	6.96	6.97	6.99	7.00	7.01	7.03	7.04
33		7.05	7.06	7.08	7.09	7.10	7.11	7.13	7.14	7.16	7.17
34		7.19	7.20	7.21	7.23	7.24	7.25	7.26	7.28	7.29	7.30
35		7.31	7.33	7.34	7.36	7.37	7.39	7.40	7.41	7.43	7.44
36		7.45	7.46	7.48	7.49	7.50	7.51	7.53	7.54	7.55	7.56
37		7.58	7.59	7.60	7.61	7.63	7.64	7.66	7.67	7.69	7.70
38		7.71	7.73	7.74	7.75	7.76	7.78	7.79	7.80	7.81	7.83
39		7.84	7.86	7.87	7.89	7.90	7.91	7.93	7.94	7.95	7.96
40		7.98	7.99	8.00	8.01	8.03	8.04	8.05	8.06	8.08	8.09
41		8.10	8.11	8.13	8.14	8.16	8.17	8.19	8.20	8.21	8.23
42		8.24	8.25	8.26	8.28	8.29	8.30	8.31	8.33	8.34	8.36
43		8.37	8.39	8.40	8.41	8.43	8.44	8.45	8.46	8.48	8.49

TABLE IV - CONTINUED

H → ↓ E										
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
44	8.50	8.51	8.53	8.54	8.55	8.56	8.58	8.59	8.60	8.61
45	8.63	8.64	8.65	8.66	8.68	8.69	8.70	8.71	8.73	8.74
46	8.75	8.76	8.78	8.79	8.80	8.81	8.83	8.84	8.85	8.86
47	8.88	8.89	8.90	8.91	8.93	8.94	8.95	8.96	8.98	8.99
48	9.00	9.01	9.02	9.03	9.04	9.06	9.07	9.08	9.09	9.10
49	9.11	9.13	9.14	9.15	9.16	9.18	9.19	9.20	9.21	9.22
50	9.23	9.24	9.26	9.27	9.28	9.29	9.30	9.31	9.33	9.34
51	9.35	9.36	9.38	9.39	9.40	9.41	9.42	9.43	9.44	9.46
52	9.47	9.48	9.49	9.50	9.51	9.53	9.54	9.55	9.56	9.58
53	9.59	9.60	9.61	9.63	9.64	9.65	9.66	9.68	9.69	9.70
54	9.71	9.73	9.74	9.75	9.76	9.78	9.79	9.80	9.81	9.83
55	9.84	9.85	9.86	9.88	9.89	9.90	9.91	9.93	9.94	9.95
R	E		R	E		R	E		R	E
56	10.0		66	11.0		76	12.1		86	12.9
57	10.1		67	11.1		77	12.2		87	13.0
58	10.2		68	11.3		78	12.3		88	13.0
59	10.3		69	11.4		79	12.3		89	13.1
60	10.4		70	11.5		80	12.4		90	13.2
61	10.5		71	11.6		81	12.5		91	13.3
62	10.6		72	11.7		82	12.6		92	13.3
63	10.7		73	11.8		83	12.6		93	13.4
64	10.8		74	11.9		84	12.7		94	13.5
65	10.9		75	12.0		85	12.8		95	13.6

TABLE IV - CONTINUED

R	E	R	E	R	E	R	E
96	13.7	105	14.3	130	16.1	155	17.7
97	13.7	110	14.75	135	16.4	160	18.0
98	13.8	115	15.1	140	16.7	165	18.3
99	13.9	120	15.4	145	17.1	170	18.6
100	14.0	125	15.75	150	17.4		

TABLE V

ESCAPE CORRECTION FACTOR FOR SINGLE ALPHA PARTICLES TRACKS (C_e)

$$C_e = \left[1 - \frac{L}{2(t - 4)} \right]^{-1},$$

where t = emulsion thickness in microns
 L = track length in microns *

E → ↓ C_{ey}	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	1.017	1.018	1.019	1.020	1.021	1.022	1.023	1.027	1.029	1.031
2	1.033	1.036	1.038	1.040	1.041	1.044	1.046	1.049	1.052	1.054
3	1.056	1.059	1.061	1.064	1.066	1.069	1.072	1.075	1.078	1.080
4	1.082	1.086	1.089	1.093	1.097	1.100	1.102	1.106	1.109	1.113
5	1.116	1.120	1.123	1.124	1.131	1.136	1.140	1.144	1.148	1.152
6	1.156	1.160	1.164	1.170	1.175	1.180	1.184	1.190	1.195	1.214
7	1.204	1.210	1.215	1.221	1.227	1.234	1.240	1.246	1.252	1.259
8	1.265	1.271	1.277	1.284	1.290	1.298	1.305	1.312	1.318	1.326
9	1.333	1.341	1.348	1.354	1.360	1.370	1.380	1.388	1.396	1.406
10	1.415	1.423	1.431	1.442	1.452	1.462	1.472	1.484	1.496	1.508
11	1.520	1.530	1.540	1.554	1.567	1.574	1.591	1.605	1.619	1.633
12	1.647	1.651	1.675	1.691	1.710	1.720	1.749	1.769	1.790	1.811
13	1.831	1.853	1.875	1.900	1.925	1.950	1.975	2.000	2.020	2.055
14	2.085	2.116	2.147	2.181	2.207	2.247	2.287	2.324	2.360	2.404
15	2.447	2.504	2.560	2.618	2.675	2.763	2.850	2.885	2.920	2.980
16	3.039	3.104	3.169	3.275	3.380	3.465	3.550	3.663	3.785	3.922

* t = 100 microns in the case of the plate used in this work.

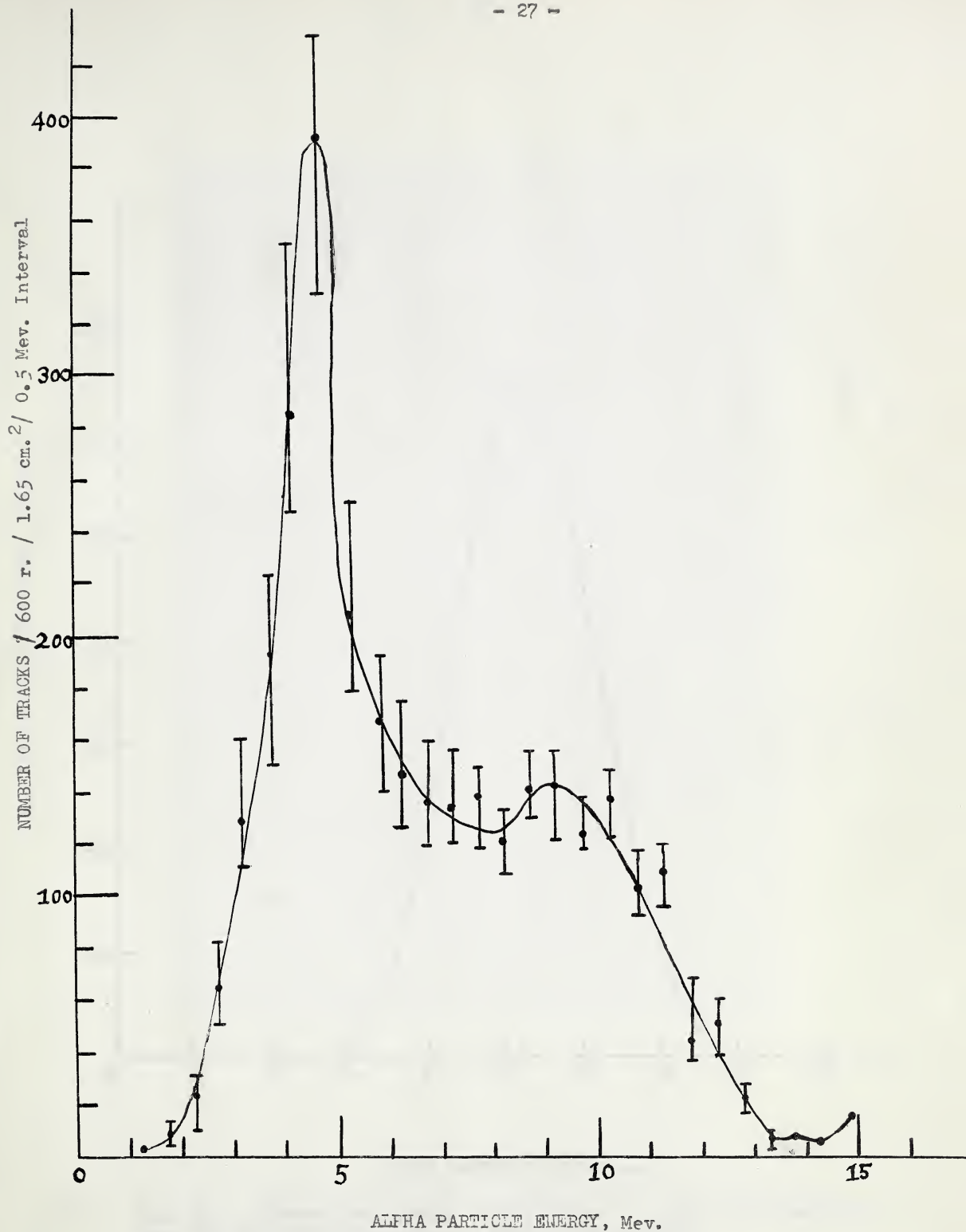


Fig. 1a. Energy spectrum of single alpha particles resulting from the photonuclear reaction of silver and bromine nuclei found in 1.65 cm.² of plate E1 (205) irradiated at maximum betatron energy 24 Mev. The spectrum has been corrected for the escape of alpha particles from the emulsion.

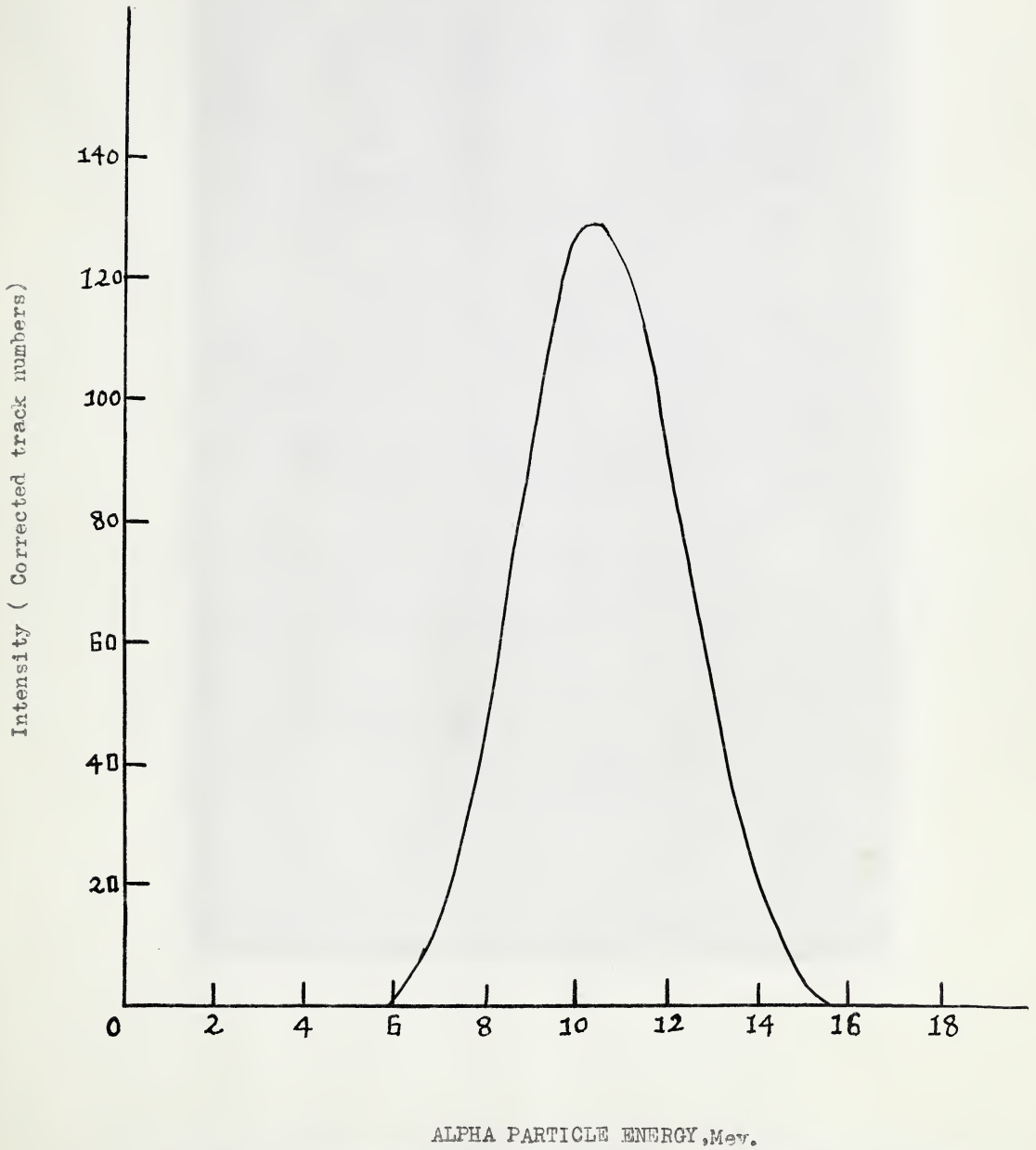


Fig. 1b. Theoretical energy distribution of single alpha particles from the photo-disintegration of silver and bromine.



Fig. II. Photograph of a single alpha particle track due to the photo-disintegration of silver and bromine nuclei, i.e., $\text{Ag}(\gamma, \alpha)$ Rh & Br(γ, α)As



Fig. III Photograph of a carbon star, due to the reaction $C^{12}(\gamma, \alpha)Be^8 \rightarrow 2He^4$ or $C^{12}(\gamma, \alpha)Be^8 \rightarrow 2He^4$. The longest alpha particle track is due to the first-emitted alpha particle. There is a momentum balance among three prongs.



Fig. IV. Photograph of a star, probably due to the reaction $N^{14}(\gamma, Li^8)2He^4$.
A short stub is from a Li^8 nucleus.



Fig. V. Photograph of an oxygen star due to the reaction $O^{16}(\gamma, 4\alpha)$.
There is a momentum balance among the prongs.

... ..



Fig. VI. Photograph of an oxygen star due to the reaction $C^{16}(\alpha, Be^8)2He^4$. The typical V-configuration of the alpha-particles from the breakup of the ground state Be^8 is visible. This event is known as birdfoot.

DISCUSSION.

This examination of a nuclear research emulsion irradiated at maximum betatron energy 24 Mev has revealed all the possible reactions involving the emission of alpha particles in all major constituents of the emulsion.

The double-peaked alpha particle energy spectrum resulting from the photo-disintegration of silver and bromine has been studied. The lower energy peak has been considered as arising from threshold-favored alpha-particle emission in reactions of the types $(\gamma, \gamma'\alpha)$, $(\gamma, \pi\alpha)$ and $(\gamma, p\alpha)$. The higher energy peak has been considered as arising from parallel and cascade reactions principally of the types (γ, α) , $(\gamma, \alpha n)$ and $(\gamma, \gamma'\alpha)$. However, the more precise interpretation of these events could have been deduced in terms of the statistical theory of nuclear reactions, if the general features of the cross-section curves had been obtained.

For a few photonuclear reactions, such as $\text{Cl}^{35}(\gamma, \alpha)\text{Be}^8 \rightarrow 2\text{He}^4$ and $\text{O}^{16}(\gamma, 4\alpha)$, an irradiation at a single energy setting of the betatron is sufficient to determine reaction cross sections as a function of photon energy, since the energies of all the disintegration products may be determined separately for each event. This procedure cannot be applied to photonuclear reactions of silver and bromine nuclei, since in these cases the alpha-particles are followed or are accompanied by gamma-rays, protons or neutrons, which have not been recorded in this experiment. It is necessary to determine the yield of alpha-particles as a function of bremsstrahlung energy by means of the photon difference method (20) in order to obtain the photo-alpha cross-section curves. Lack of information on the cross-section curves has made the interpretations highly speculative.

The energy distribution of single alpha particles did not show the secondary maxima in the region above 6 Mev as observed by Greenberg.

The events occurring in light nuclei are distinguishable from those in silver and bromine. However, if a 4 or 5 Mev alpha-particle track originates

from an oxygen or nitrogen nucleus, the track of the recoiling nucleus would have a range of less than 2 microns and might easily be misinterpreted by the observer. Therefore some of the low energy particles included in the observed spectrum may have originated from light nuclei in the emulsion.

BIBLIOGRAPHY

- (1) Nabholz, H., Stoll, P., and Waeffler, H. Helv.Phys.Acta, 12; 701, 1952.
- (2) Haslam, R.N.H., Cameron, A.G.W., Cooke, J.A. and Crosby, E.H. Can. J. Phys. 30: 349, 1952.
- (3) Miller, C.H. and Cameron, A.G.W. Can.H.Phys. 31: 723, 1953.
- (4) Blatt, J.M. and Weisskopf, V.F. Theoretical Nuclear Physics, Chap.VIII. (John Wiley and Sons, Inc., New York, 1952.)
- (5) Greenberg, L.H., Private Communication, 1954.
- (6) Millar, C.H. and Cameron, A.G.W. Phys.Rev. 78:78. 1950.
- (7) Haenni, H., Telegdi, V.L., Zuenti, W. Helv. Phys.Acta. 21:203, 1948
- (8) Telegdi, V.L. and Eder Helv. Phys Acta. 25: 55. 1952.
- (9) Goward, F.K. and Wilkins, J.J. Proc.Phys.Soc.London, A, 66:357. 1953.
- (10) Wilkins, J.J. and Goward, F.K. Proc.Phys.Soc.London, A.63:663. 1950.
- (11) Millar, C.H. and Cameron, A.G.W. Phys.Rev. 79:182.1950.
- (12) Goward, F.K. and Wilkins, J.J. Proc. Phys.Soc.London, A, 63:1171. 1950.
- (13) Goward, F.K. and Wilkins, J.J. Proc.Phys.Soc. London, A, 64:94. 1951.
- (14) Goward, F.K. and Wilkins, J.J. Proc.Phys.Soc. London, A, 64:312, 1951.
- (15) Toldhaber, M. and Teller, E. Phys.Rev. 80:407, 1950.
- (16) Katz L. and Cameron, A.G.W. Phys.Rev. 84:115, 1951.
- (17) Weisskopf, V.F. and Ewing, D.H. Phys. Rev. 57:472, 1940.
- (18) Cameron, A.G.W. and Katz, L. Phys. Rev. 84:608, 1951.
- (19) Millar, C.H. Can. J. Phys. 31:262, 1953.
- (20) Katz, L. and Cameron, A.G.W. Can. J. Phys. 29:518, 1951.
- (21) Livingston, M.S. and Bethe, H.A. Revs. Modern Phys. 9, 245 (1937).
- (22) Smith, J.M. Phys.Rev. 71, 32 (1946).
- (23) Webb, J.H. Phys.Rev. 74, 511. 1948.
- (24) Lathes, C.M.G., Fowler, P.M. and Cueer, P. Proc.Phys.Soc.London, A, 59: 885, 1950.
- (25) Bradner, F.M., Smith, W.H., Barkas, and A.S.Bishop, Phys.Rev. 77, 462, 1950.
- (26) Yagoda, H. Radioactive measurement with nuclear emulsion. John Wiley and Sons, Inc., N. Y., 1949

Appendix

1. The first part of the report is devoted to a general survey of the situation in the country.	(1)
2. The second part contains a detailed description of the economic situation.	(2)
3. The third part deals with the social and cultural aspects of the situation.	(3)
4. The fourth part discusses the political situation and the role of the government.	(4)
5. The fifth part contains a summary of the findings and conclusions of the study.	(5)
6. The sixth part lists the sources of information used in the study.	(6)
7. The seventh part contains a list of abbreviations and symbols used in the report.	(7)
8. The eighth part contains a list of references to other works on the same subject.	(8)
9. The ninth part contains a list of names of persons mentioned in the report.	(9)
10. The tenth part contains a list of names of institutions mentioned in the report.	(10)
11. The eleventh part contains a list of names of places mentioned in the report.	(11)
12. The twelfth part contains a list of names of dates mentioned in the report.	(12)
13. The thirteenth part contains a list of names of events mentioned in the report.	(13)
14. The fourteenth part contains a list of names of organizations mentioned in the report.	(14)
15. The fifteenth part contains a list of names of publications mentioned in the report.	(15)
16. The sixteenth part contains a list of names of persons mentioned in the report.	(16)
17. The seventeenth part contains a list of names of institutions mentioned in the report.	(17)
18. The eighteenth part contains a list of names of places mentioned in the report.	(18)
19. The nineteenth part contains a list of names of dates mentioned in the report.	(19)
20. The twentieth part contains a list of names of events mentioned in the report.	(20)
21. The twenty-first part contains a list of names of organizations mentioned in the report.	(21)
22. The twenty-second part contains a list of names of publications mentioned in the report.	(22)
23. The twenty-third part contains a list of names of persons mentioned in the report.	(23)
24. The twenty-fourth part contains a list of names of institutions mentioned in the report.	(24)
25. The twenty-fifth part contains a list of names of places mentioned in the report.	(25)
26. The twenty-sixth part contains a list of names of dates mentioned in the report.	(26)
27. The twenty-seventh part contains a list of names of events mentioned in the report.	(27)
28. The twenty-eighth part contains a list of names of organizations mentioned in the report.	(28)
29. The twenty-ninth part contains a list of names of publications mentioned in the report.	(29)
30. The thirtieth part contains a list of names of persons mentioned in the report.	(30)

APPENDIX I

Calculation of Range and Energy.

The high concentration of silver and bromine in nuclear research emulsions causes a considerable reduction in their thickness after fixation, when the unused silver is removed. If the ratio between the thickness of the emulsion before and after processing, which is called shrinkage factor, is known, it is possible to correct for this effect in determining track lengths by using the formula

$$R = \left[Y^2 + (SZ)^2 \right]^{\frac{1}{2}}$$

where R = the original track length (before processing)

Y = horizontal component of a track (after processing)

Z = vertical component of a track (after processing).

S = shrinkage factor.

The horizontal component of a track was measured by means of a micrometer disc attached to the microscope eyepiece and the vertical component of a track was measured by the vertical movement of the microscope tube. The shrinkage factor 2.63 was used for this particular plate.

The shrinkage factor of an emulsion may be determined in the following way: The plate is exposed to a nearly point source of alpha-particles at a distance, D , from the emulsion. The position of the track with respect to the position of the source determines the angle of the tracks. The horizontal and vertical components of a track in the developed emulsion are measured (ℓ and d respectively) and the distances L and D are obtained. (Figure VII. A plot of ℓ/d against L for several tracks yields a straight line if S is constant, and the value of S may be determined from the slope of the line.

The theoretical range-energy relationship for protons in air was computed by Livingston and Bethe (21), and Smith (22). From this relationship for protons the range-energy relationship for alpha-particles in air was determined by Webb (23). These relationships may be applied to nuclear emulsions by multiplying the range values by the stopping power of the emulsion relative to air.

The range-energy relationship for proton and alpha particles has also been determined experimentally by a number of investigators. Lattes, Fowler and Cueer (24), and Bradner et al. (25) have published the most comprehensive results. The former workers employed Ilford type B1 emulsions and the latter C2 emulsions in their work. Cameron altered the values of the former workers by 3% to take into account the greater stopping power of B1 emulsions. This relationship was re-examined before adopting it in this experiment.

The following is a list of the names of the persons who

have been appointed to the various committees of the

Association, and the names of the persons who have been

appointed to the various committees of the Association,

and the names of the persons who have been appointed to

the various committees of the Association.

The following is a list of the names of the persons who

have been appointed to the various committees of the

Association, and the names of the persons who have been

appointed to the various committees of the Association,

and the names of the persons who have been appointed to

the various committees of the Association.

The following is a list of the names of the persons who

have been appointed to the various committees of the

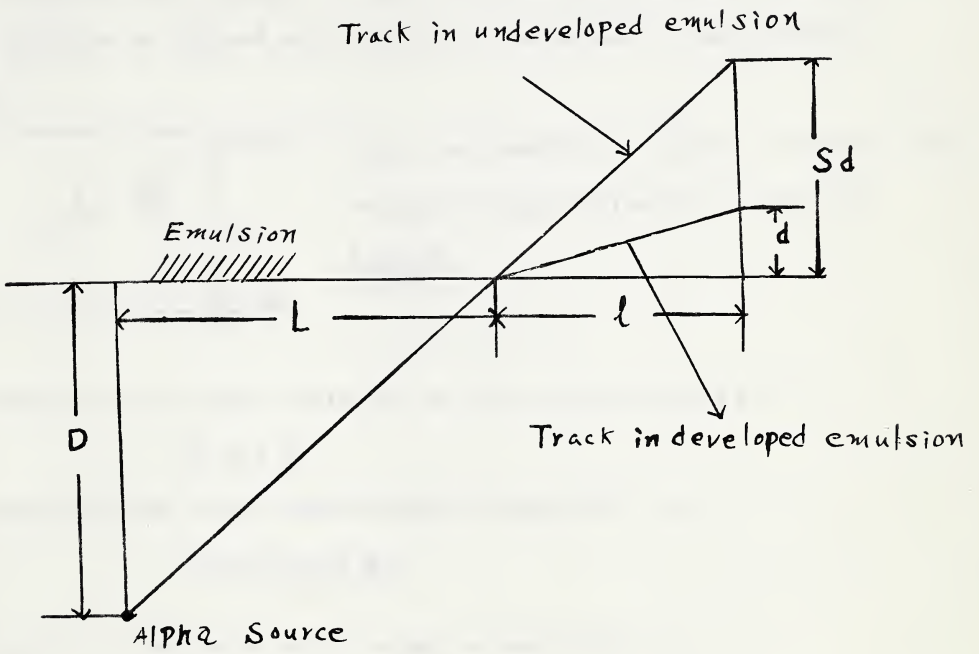
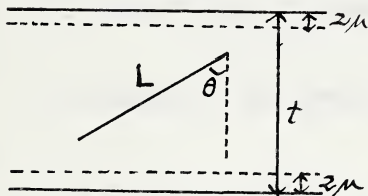


Fig. VII Diagram showing a method of determining the shrinkage factor for an emulsion.

APPENDIX II

Calculation of Escape Correction Factor

Consider an emulsion of thickness t microns, and assume that single alpha particle tracks of length L microns are randomly oriented in the emulsion at angle θ with respect to the normal to the emulsion surface



Then, the fraction of tracks of length L that escape at angle θ from the emulsion is

$$\frac{L \cos \theta}{(t - L)},$$

The fraction of the tracks contained in solid angle $2\pi \sin \theta d\theta$ is $\sin \theta d\theta$.

The fraction of the tracks which escape at angle θ is

$$\frac{L \cos \theta \sin \theta d\theta}{(t - L)}$$

The fraction of the tracks which escape at any angle is

$$\int_0^{\pi/2} \frac{L \cos \theta \sin \theta d\theta}{(t - L)} = \frac{L}{2(t - L)}$$

Therefore the escape correction factor by which the observed number of tracks of length L within the emulsion must be multiplied in order to obtain the total number of tracks which originate in the emulsion is given by

$$C_e = \left[1 - \frac{L}{2(t - L)} \right]^{-1}$$

APPENDIX III

Identification of the momentum balance among the three prongs of a carbon star.

Consider a particle of mass $3m$ which breaks into three equal particles each of mass m , of momenta \vec{P}_1 , \vec{P}_2 and \vec{P}_3 , and of energies E_1 , E_2 and E_3 .

$$P = \sqrt{2mE}$$

$$\text{By conservation of momentum } \vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0.$$

The energy of the photon involved in the reaction is given by

$$h\nu = E_1 + E_2 + E_3 + \text{the binding energy.}$$

Example:

A carbon star in a single plane located at (21.2: 121.5: 7.0)

	Range in μ	Energy in Mev
1	12.7	3.61
2	4.23	1.27
3	5.64	1.76

$$h\nu = 3.61 + 1.27 + 1.76 + 7.15 = 13.8 \text{ Mev}$$

$$P_1 = \sqrt{2 \times 4 \times 3.61} = \sqrt{28.88} = 5.38.$$

$$P_2 = \sqrt{2 \times 4 \times 1.27} = \sqrt{10.16} = 3.18$$

$$P_3 = \sqrt{2 \times 4 \times 1.76} = \sqrt{14.08} = 3.76$$

PROBLEM 1

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation

$$f(x+y) = f(x) + f(y) \quad \forall x, y \in \mathbb{R}.$$

Assume that f is continuous at $x=0$. Show that f is linear, i.e. $f(x) = cx$ for some constant c .

Hint: First show that $f(0) = 0$ and that f is additive, i.e. $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

$$f(0) = f(0+0) = f(0) + f(0) \implies f(0) = 0.$$

$$f(x) = f(x+0) = f(x) + f(0) = f(x) + 0 = f(x).$$

Next, show that f is additive, i.e. $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

$$f(x+y) = f(x) + f(y) \quad \forall x, y \in \mathbb{R}.$$

$$f(x) = f(x+y) - f(y) = f(x) + f(y) - f(y) = f(x).$$

□

Now, let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation

$$f(x+y) = f(x)f(y) \quad \forall x, y \in \mathbb{R}.$$

$$f(0) = f(0+0) = f(0)f(0) \implies f(0) = 0 \text{ or } f(0) = 1.$$

$$\text{If } f(0) = 0, \text{ then } f(x) = f(x+0) = f(x)f(0) = 0 \text{ for all } x \in \mathbb{R}.$$

$$\text{If } f(0) = 1, \text{ then } f(x) = f(x+0) = f(x)f(0) = f(x) \text{ for all } x \in \mathbb{R}.$$

$$f(x+y) = f(x)f(y) \implies f(x) = \frac{f(x+y)}{f(y)} = \frac{f(x)f(y)}{f(y)} = f(x).$$

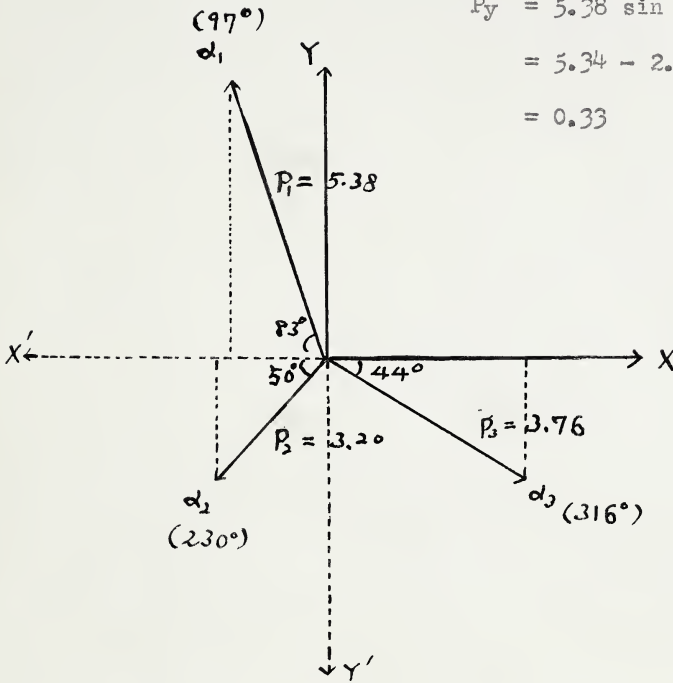
$$f(x) = \frac{f(x+y)}{f(y)} = \frac{f(x)f(y)}{f(y)} = f(x).$$

$$f(x) = \frac{f(x+y)}{f(y)} = \frac{f(x)f(y)}{f(y)} = f(x).$$

$$f(x) = \frac{f(x+y)}{f(y)} = \frac{f(x)f(y)}{f(y)} = f(x).$$

$$\begin{aligned} P_x &= - 5.38 \cos 83 - 3.18 \cos 50 + 3.76 \cos 44 \\ &= - 0.645 - 2.04 + 2.70 \\ &= + 0.01 \end{aligned}$$

$$\begin{aligned} P_y &= 5.38 \sin 83 - 3.18 \sin 50 - 3.76 \sin 44 \\ &= 5.34 - 2.44 - 2.61 \\ &= 0.33 \end{aligned}$$



B29771